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THE USE OF PROBABILISTIC TECHNIQUES IN THE GENERATION OF AN OPTIMISED INSPECTION STRATEGY

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1. INTRODUCTION

This document is a review of Reference 1 'Probability Based Fatigue Inspection Planning and Repair' which sets out a probabilistic method of inspection planning by the examination of an example K joint and the derivation of a reliability index for this component. The variations of the reliability index with time and its modification due to inspection is also quantified.

The full review of Reference 1 follows a section which describes the basic concepts involved in the probabilistic reliability approach.

These ideas are then put in the context of the established deterministic method described in Reference 2 for the development of an optimised inspection strategy. A probabilistic approach is proposed which can be used for structural components identified as critical by the deterministic method. This probabilistic method relies on the determination of the probability of failure of a component and a comparison with the corresponding acceptable probability of failure following Reference 5 (CIRIA Report UR9).

Finally a scope of work is suggested for the implementation of these ideas.

2. BACKGROUND

2.1 Probabilistic Techniques

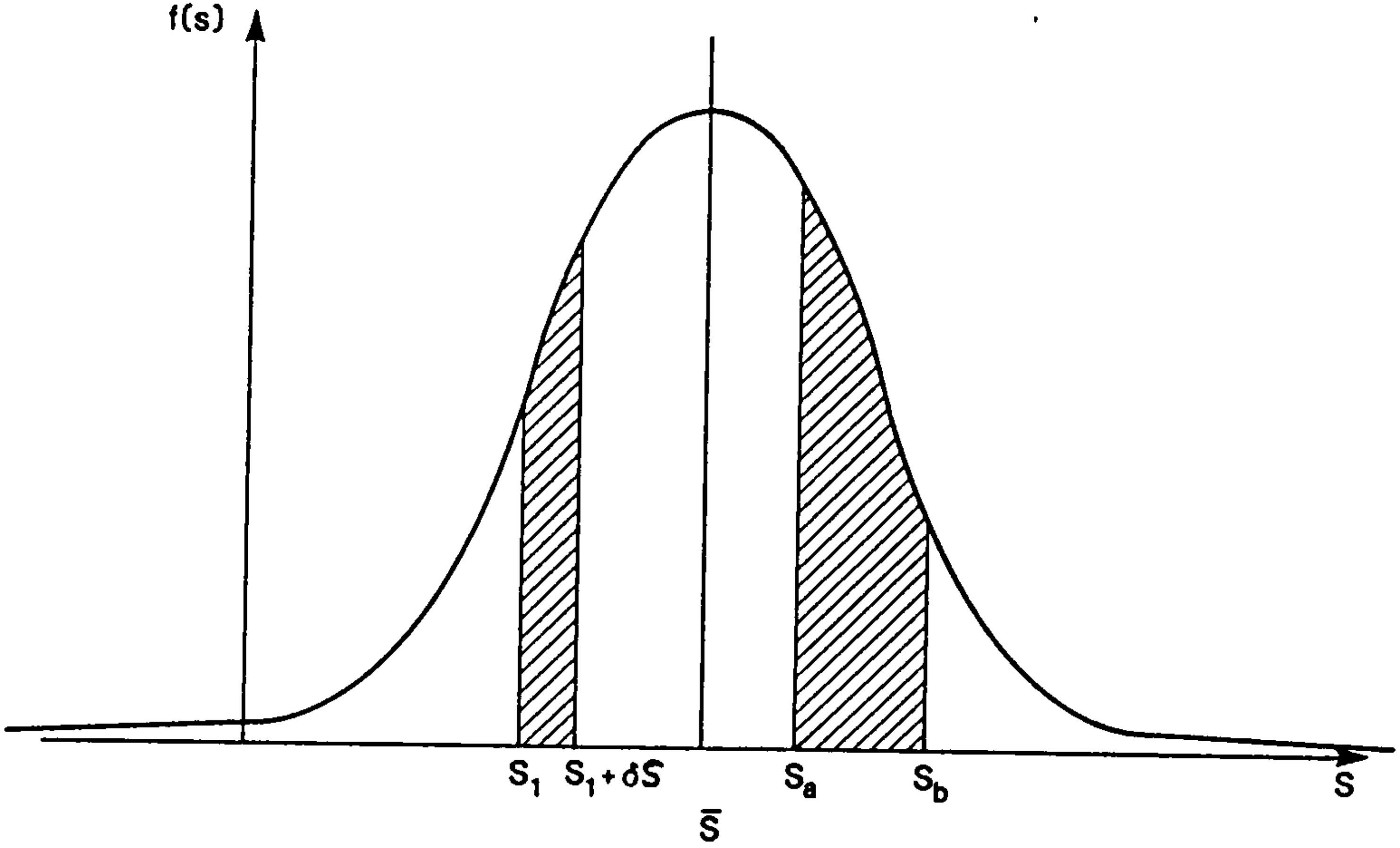
For each physical parameter 's' such as total load on a member or stress in a hot-spot a value s may be measured at a particular time. A large number of observations of s under varying load conditions will give rise to a 'probability distribution' of s F(s) shown in Figure 1. The value of F(s) is an indication of the probability that s lies near a particular value. More correctly, the probability that s lies in the interval s and s + δs is given by $F(s_1)\,\delta s$ the area indicated. The total area under the curve is 1 indicating the total probability that S lies between + and $-\infty$ the mean value $\bar{s}$ is also indicated in Figure 1.

The probability that s lies between sa and sb is also indicated on the Figure and is given by

$$P_r(s_a < s < s_b) = \int_{S_a}^{S_b} f(s)\,ds$$

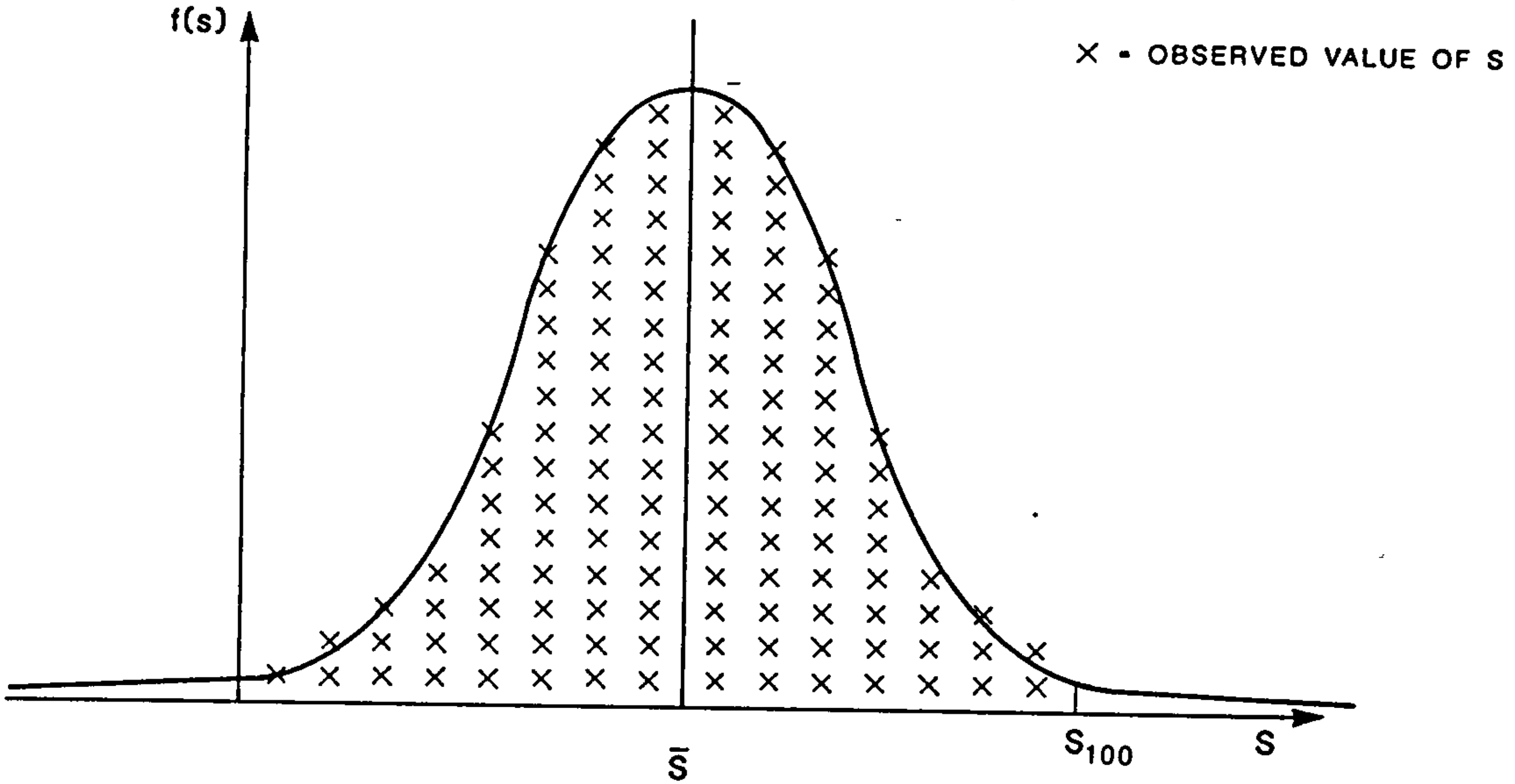
The mean value or expected value is thus a measure of the position of the probability density function (p.d.f.) curve f(s). A deterministic evaluation of s for design purposes may be taken as that value of s which is exceeded once every 100 years ie. with a probability of exceedence of 0.01 per year which is the area of the shaded region in Figure 2.

The position of S_{100} and its distance from the mean value $\bar{S}$ is determined by the standard deviation of the probability distribution 6_s. The larger 6_s the greater the spread.



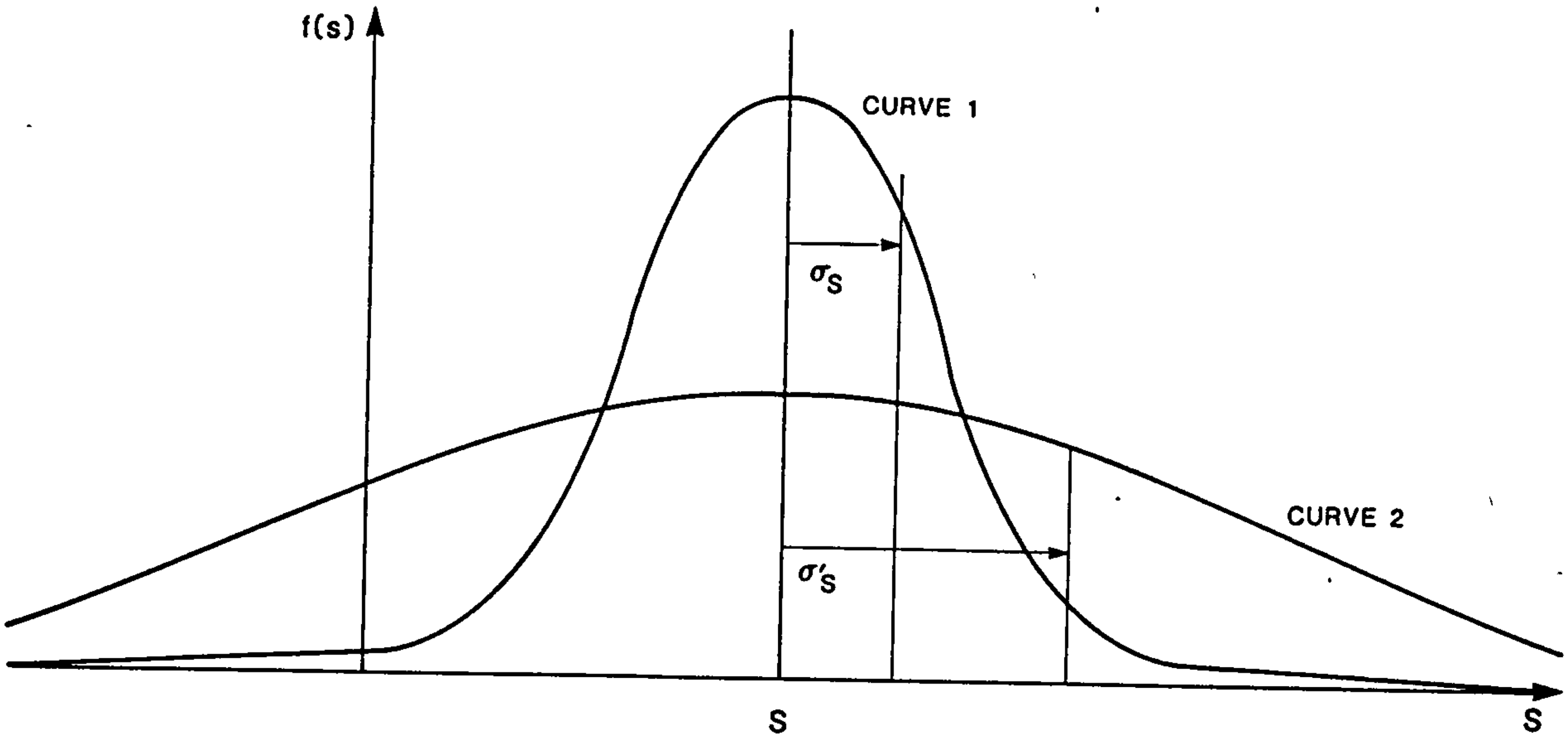
PROBABILITY DENSITY FUNCTION f(s)

FIG. 1



100 YEAR RETURN PERIOD DESIGN VALUE S_{100}

FIG. 2



THE STANDARD DEVIATION σ_S

FIG. 3

Curve 2 has a larger standard deviation than the more peaked curve 1. (For a normal distribution that value with 0.01 probability of exceedence is 2.3263 standard deviations above the mean value $\bar{S}$ ie.

$$S_{100} = \bar{S} + 2.3263\,\sigma_s$$

(for a p.d.f., F(s) representing 1 year maximum values)

The coefficient of variation C.O.V. C_s is defined as

$$C_S = \frac{\sigma_s}{\bar{S}} = \frac{\text{Standard Deviation}}{\text{Mean}}$$

which is a non-dimensional measure of the degree of spread.

The number of standard deviations E by which the observed or chosen value of the random variable s exceeds the mean $\bar{s}$ is a measure of the probability of exceedance of that value (see Table 1). A 'confidence level' can be associated with the probability of exceedance of an observed value of s if the probability density or distribution function f(s) and the standard deviation for the distribution is known.

Because the value of the random variable S is commonly the result of a large number of independent contributing factors the form of f(s) can often be assumed to be the Normal distribution given by:

$$f(s) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(\bar{s}-s)^2}{2\sigma_s^2}}$$

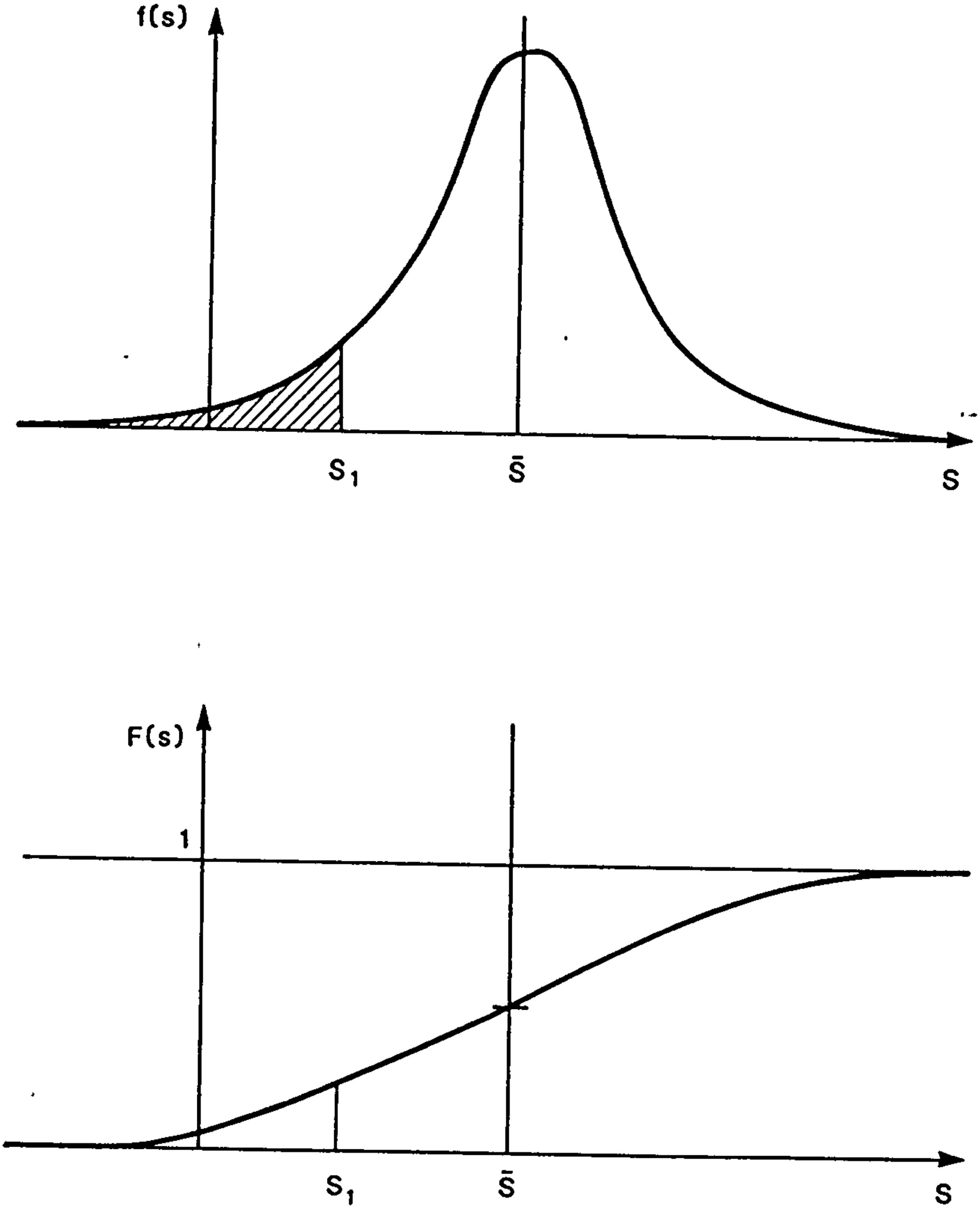
Other commonly assumed distributions are Log-normal Exponential, Wiebull and Rayleigh. If the Normal distribution is assumed then probabilities of exceedance may be associated with a given number of standard deviations from the mean as below:

TABLE + NORMAL DISTRIBUTION	
Probability of Exceedance	E Number of Standard Deviations above Mean
.500	0
.150	1.0364
.022	2.0142
.001	3.0902
.05 (5%)	1.6449
.02 (2%)	2.0537
.01 (1%)	2.3263

For values at 1.6449 standard deviations above the mean there is a 5% chance of exceedance the 95% confidence limit is associated with this value as there is a 95% chance that this value will not be exceeded.

Another useful function in statistical analysis is the cumulative probability distribution function or 'Distribution Function' F(s).

If a function is generated which represents the area under the curve F(s) and to left of a given value of s then this function is the cumulative probability distribution function F(s) (see Figure 4). The value F(s) is the probability that S is less than (or equal to) an observed value s. If F(s) is the standard normal distribution (with zero mean and unit standard deviation) then this function is written as $\Phi(s)$.



THE CUMULATIVE PROBABILITY
DISTRIBUTION FUNCTION F(s)

FIG. 4

Generally:

$$F(s_1) = \int_{-\infty}^{s_1} f(s)\,ds$$

2.2 Reliability Theory

Consider the load carrying capacity of a particular member r of an offshore structure. (See Figure 5 from Reference 2).

The actual strength of the component may probabilistically be represented by the right hand curve in the figure. The actual load S is also a random variable represented by the left hand curve f(s) plotted on the same axes.

The probability of failure P_f where the applied load S is greater than the actual capacity or resistance r is 'indicated' by the overlap between the two curves. In fact Pf is given by

$$P_f = P_r(r-s \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_s(x)\,dx$$

The reliability R or the probability that the member will survive the applied load S is given by

$$R = 1 - P_f$$

If we make the substitution

$$m = r - S$$

then the probability of failure is

$$P_f = P_r(m \leq 0)$$

If r and s are assumed to be normally distributed then the mean value of m is given by

$$\bar{m} = \bar{r} - \bar{s}$$

and the standard deviation of m is given by

$$\sigma_m = \sqrt{\sigma_r^2 + \sigma_s^2}$$

and m is 'normally distributed' as in Figure 6.

Then $\frac{m-\bar{m}}{\sigma_m}$ is distributed according to the standard normal distribution and the probability of failure Pf is given by

$$P_f = P_r(m \leq 0) = \Phi\left(\frac{0-\bar{m}}{\sigma_m}\right)$$

If we now define the reliability index ß by

$$\beta = \frac{\bar{m}}{\sigma_m} \quad \text{or} \quad \bar{m} = \sigma_m \beta \quad \text{or} \quad \beta = \frac{1}{COV_m}$$

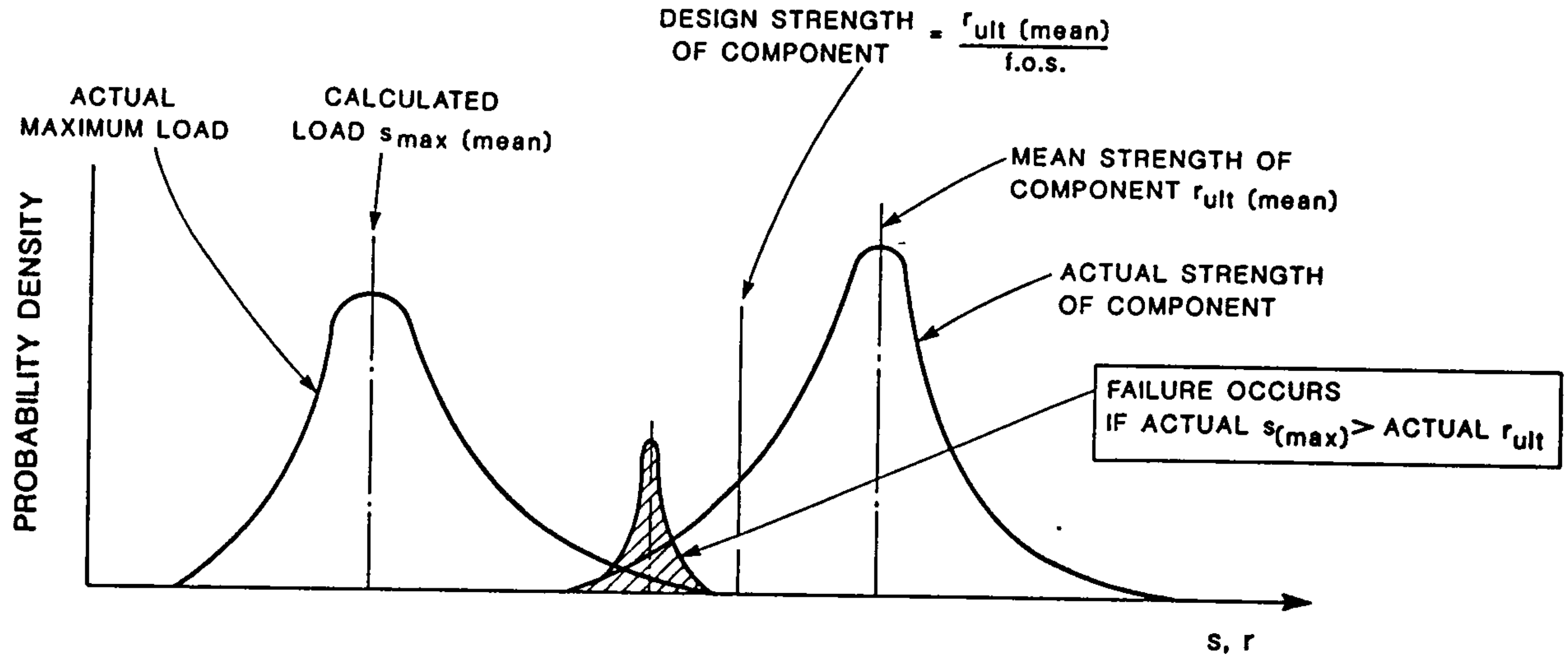
(the number of standard deviations that the mean of m is above zero)

Then the probability of failure of the component Pf is given by

$$P_f = \Phi(-\beta)$$

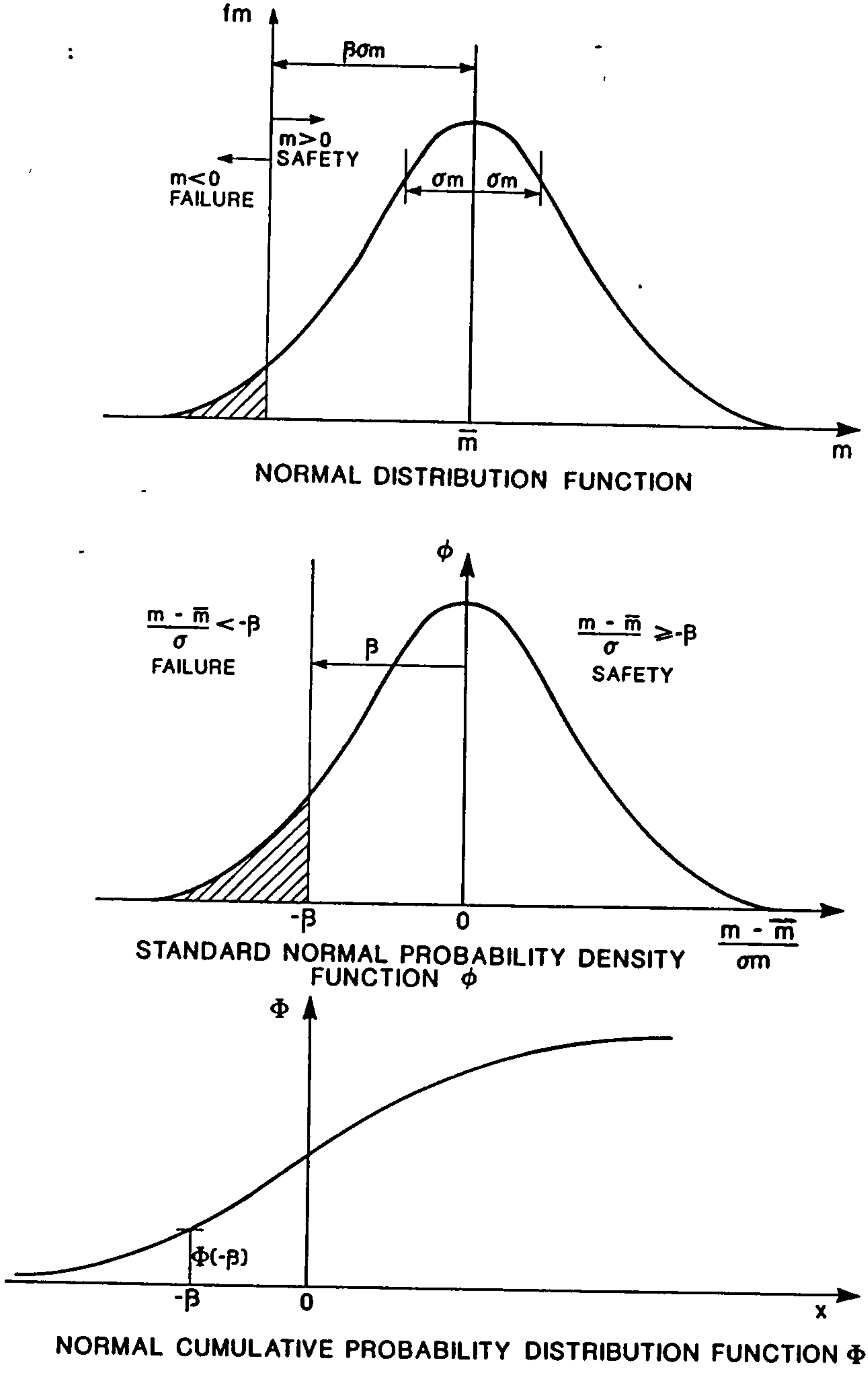
The higher ß is, the smaller the probability that the component will fail. As $\beta = \frac{\bar{r} - \bar{s}}{\sqrt{\sigma_r^2 + \sigma_s^2}}$ it is a non-dimensional measure of the separation of the two probability density functions of the resistance (r) and load variables concerned.

The partial sensitivity factor $\frac{\partial \beta}{\partial S}$ is a measure of the rate of change of ß with variations of the variable S. Figure 7 is a graph of ß against S, clearly the steeper the curve the more sensitive ß is to changes in S. $\partial\beta/\partial S$ is a measure of the slope of the ß curve. As ß may be a function of other variables the partial differential is used to separate out the effect of change in any particular variable.



RELATIONSHIP BETWEEN LOAD CARRYING CAPACITY AND LOAD IN A COMPONENT

FIG. 5



THE RELIABILITY INDEX β

FIG. 6

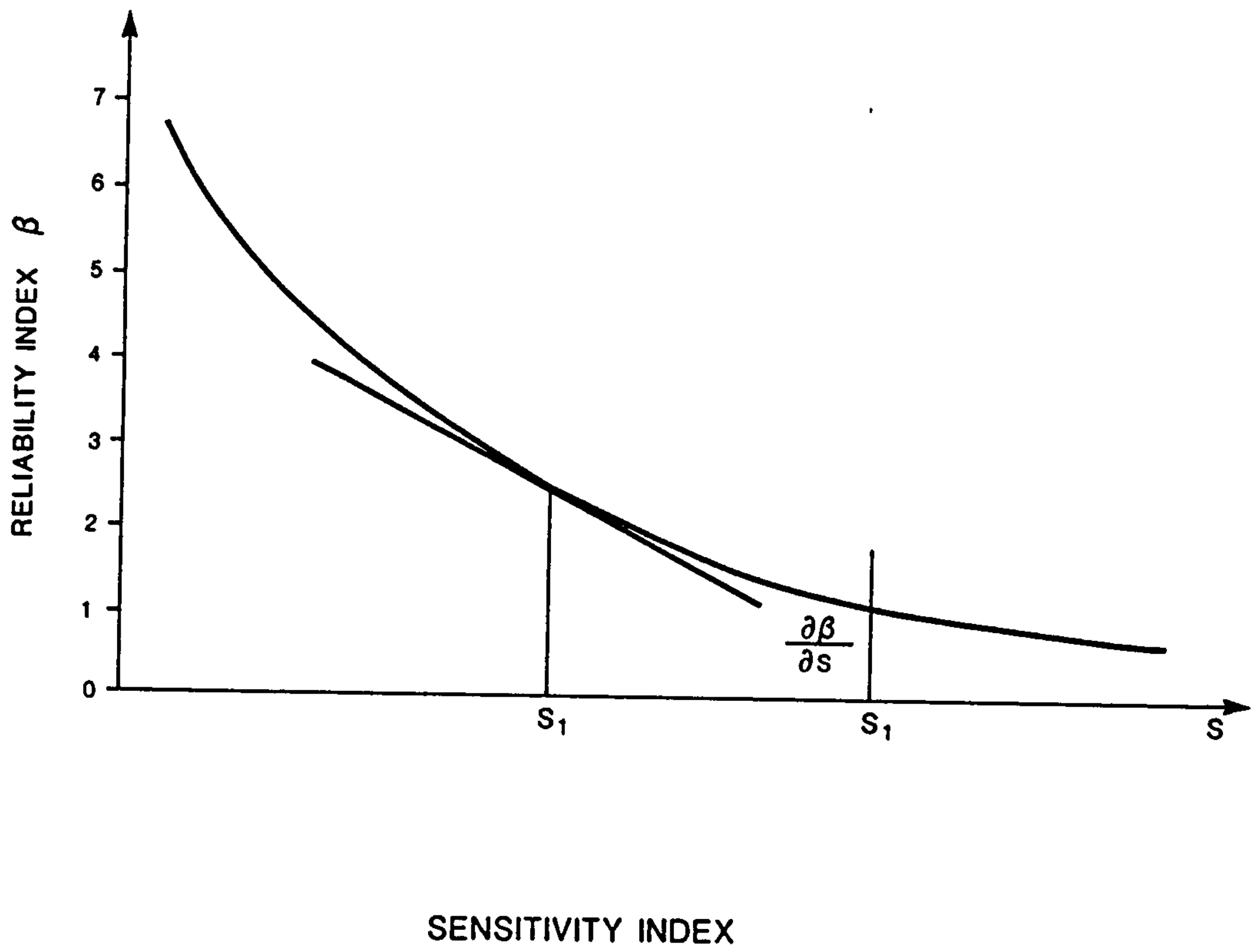


FIG. 7

3. COMMENTARY

This section is an attempt to explain in practical terms the meaning and applicability of the draft report [1]. The subsections below refer to the main parts of the paper.

The main objectives described have largely been achieved ie.:

1. To demonstrate the applicability of probabilistic methods to inspection planning.

2. To ascertain the importance/sensitivity of the principal input parameters for the probabilistic analysis.

3. To investigate the effect of different inspection regimes on the reliability of a structural component.

4. To identify shortcomings in currently available probabilistic methods for application to real structures.

Here the refefence to 'system reliability capabilities' is a reflection of the inability of reliability analysis to fully take into account the redundancy present in a typical fixed offshore jacket and the knock-on effects of failure of a node on the surrounding structural components.

3.0 Summary

This section describes the test case examined ie. a hot spot on a tubular joint examined in both the conventional (S/N curve) and fracture mechanics way.

The S/N curve is the hot spot stress range against number of cycles to failure curve. The probabilistic approach in the fatigue calculation relates to the fact that the loading is a sea state spectrum consisting of a number of cycles of loading at a range of frequencies.

In the fracture mechanics approach uncertainties in the material parameters, stress concentration factors and crack length are introduced to give a probabilistic approach.

The S/N curve is identified as the most important source of uncertainty for the conventional fatigue analysis. In the fracture mechanics analyis the material parameter is identified as most crucial.

As before the areas in need of investigation are identified as:

1. The reliability of inspection methods.

2. The acceptable probabilities of failure of components.

3. The importance of each structural component in the reliability of the total structure.

3.1 General Overview/Introduction to the Probabilistic Approach

This section is a general discussion of the use of probabilistic methods in assessing the conservatism inherent in the codes.

The concept of a limit state function is introduced. This function of the design variables (such as loads and material properties) defines the boundary between the safe and failure regions defined by various values of the design variables. From this limit state function modelled uncertainty in the design variables will give rise to a measure probability of failure using reliability methods. The total uncertainty may be evaluated and the critical, most sensitive, design variables identified.

A cautionary note is offered which warns that the probability of failure is a changing quantity which depends on the quality of information available. Failure due to gross error taken to mean errors made in design is mentioned as the major cause of failure, 90% of all failures are attributed to this cause.

3.2 Review of Current Data on Reliability of Inspection Methods

This section is largely self-explanatory. The probability of detection (POD) is identified for a number of different inspection techniques. POD curves are given for MPI and visual inspection for different crack depths. An aspect ratio depth/length of 0.15 is assumed. The uncertainty in the probability of detection is later represented as an uncertainty in the observed crack length.

3.3 Presentation of Case Study

The four sections of the case study are listed:

1. Deterministic fatigue analysis
2. Probabilistic fatigue analysis
3. Probabilistic analysis of fatigue crack growth
4. Effects of inspection on the reliability index

3.3.1 Deterministic S-N Fatigue Analysis

This is in fact what is conventionally referred to as a spectral fatigue analysis. The magnitudes of the material property parameters load tansfer functions and stress concentration factors are assumed to be known and in this sense it is a deterministic analysis. In the formulation a directional wave spectrum representing the spread of wave energy with frequency ω and wave direction θ is used. (3.1).

The next step is to derive the relationships between member end forces and moments for waves of unit height of varying frequency w and direction $H\eta Fi(w)$. The member end force spectra are then given by.

$$S_{F_i}(\omega) = H_{\eta F_i}(\omega) H^*_{\eta F_i}(\omega) S_\eta(\omega)$$

This is a measure of the spread of member end force (or moment) Fi with wave frequency w. See Reference 6 for details of the spectral method. The hot spot stresses 'σ' round the chord side of the brace to chord weld are given by the influence coefficients or stress concentration factors Ii which represent the combined effect of axial and bending loads with

$$\sigma = \sum_{i=1}^{15} I_i F_i$$

Hence the hot spot stress spectrum is given by equation (3.4).

The fatigue strength in given by the S-N curve N being the number of cycles to failure at stress range S, given by expressions 3.5 and 3.6.

The fatigue damage ΔDi caused by a single cycle at stress range si is given by

$$\Delta D_i = \frac{1}{N(S_i)}$$

following miners rule.

The total damage given by equation (3.8) is simply the summation, of damage over all sea states in all directions over the time period T for the hot spot considered. If failure is assumed to occur when

$$D = 1$$

then the fatigue life may be determined. In the language of Reliability theory we may define a safety margin function

$$f = 1 - D$$

which indicates failure when $f \leq 0$ which is the failure criterion.

3.3.2 Probabilistic S-N Fatigue Analysis

This analysis uses the same method as the 'deterministic' method described above except that the analysis parameters are considered to have some uncertainty associated with their values. This includes the damage at failure Δ which is assumed to have a 20% coefficent of variation (COV).

The failure criterion is

$$\Delta - D \leq 0$$

The probability of failure Pf is the probability that this failure criterion is satisifed. In the usual way (for a normally distributed random variable) the reliability index β may be calculated as shown in Section 2.2.

$$P_f = \Phi(-\beta) \quad or \quad \beta = -\Phi^{-1}(P_f)$$

$\beta = 1.4$ is the level assumed by the standard requirement that $\Delta \leq 1$.

Table 3.4 shows the relative importance of the uncertainties associated with different aspects of the analysis.

The sensitivity factor associated with changes in the load coefficients is given and hence the expected change in the reliability index ß with the COV of these coefficients is simply dervied from.

$$\Delta\beta_R \simeq \frac{\partial\beta_R}{\partial V_L}\Delta V_L$$

This formulation also enables the effect on ß of neglecting the COV of the random load coefficient to be determined thus quantifying the effects of uncertainty in the contributing factors to the failure criterion.

3.3.3 Probabilistic Fracture Mechanics Study

As the S/N curves were found to be the main source of uncertainty in determining the fatigue life (or defining the failure criterion) this section deals directly with the failure criterion of the existence of a through thickness crack by considering the mechanics of crack growth directly. A crack depth to length ratio of 0.15 is assumed and the Paris Law giving the rate of crack depth extension $\frac{da}{dN}$ with material parameters C and M.

Viz: $$\frac{da}{dN} = C(\Delta K)^m$$

Δk is the stress intensity range which depends on the ambient stress range $\Delta\sigma$ and geometry function Y(a) which represents stress concentration and sometimes through thickness variation of stress as the crack progresses. Here $\Delta\sigma$ is the hot spot stress range as used in the conventional fatigue analysis. A further factor Mk is sometimes applied to represent the stress raising effect of local notches in the weld profile.

Assumed forms of Y and Mk are given in equations 3.1.5.

Integrating the Paris Law with x replacing a as the variable of integration and re-writing $\Delta\sigma$ as Si, the stress range for cycle i equation 3.16 is obtained with ao being the initial defect depth and a 'a' being the current crack depth.

At failure $a = a_c$ giving a failure criterion

$$M = \int_{a_0}^{a_c} \frac{dx}{(Y(x))^m \, (\pi x)^{m/2}} - C \sum_{i=1}^{N} S_i^m \leqslant 0$$

and a probability of failure Pf given by:

$$P_f = P(M \leqslant 0)$$

The initial crack size a_0 may now be treated as a random variable as may be geometry function Y and the material constant C, m is assumed fixed.

Table 3.5 then shows the relative importance of the various uncertainties in the input variables.

3.3.4 Effect of Inspection Procedures

Instead of treating the stress range S as a complex combination of environmental and response parameters the stress range is treated is being distributed according to a Weibull model with composite parameters A and B. The paper does not fuly describe how A and B are chosen but with the equation given 3.23 the probability distribution of S is defined in terms of the new parameters A and B with good agreement with the original distribution. Equation 3.24 shows the equivalent expression for damage $C\sum_{i=1}^{N} S_i^m$ in terms of A and B.

Equations 3.25 and 3.26 represent the situations where during inspection a crack is not detected or where a crack of length Aj is detected at time Tj. The smallest detectable crack size A_{di} is a random variable with a probability distribution function given by the POD function given earlier.

The event margin Mi is an indication of the damage which can be sustained before a fatigue crack of detectable size can be expected. The event margin Mi is defined by equation 3.28 which is a statement that the damage experienced is sufficient to produce the observed crack size Aj.

Equation 3.29 means in words

P_F^u = the probability that $M \leq 0$ given ($M_1 \leq$ and $M_2 \leq 0$ and .... and $M_r \leq 0$)

ie. the probability that no defect is observed given that no defect has been observed in the previous r inspections. Conditional probabilities of this type obey the following rule.

$$P_r(M \leq 0 \mid M_1 \leq 0) = \frac{P_r(M \leq 0 \cap M_1 \leq 0)}{P_r(M_1 \leq 0)}$$

This rule gives rise to equation 3.30.

The corresponding expression for when a crack is found is given in the appendix.

If repairs are undertaken then the updated failure probability is given by equation 3.33 for the new 'event margin' defined in 3.32.

Hence in this way the failure probability may be updated following the results of an inspection.

If the probability of failure P_f (and hence the reliability index) is evaluated in this way then it is possible to identify the inspection periods which will keep the probability of failure above a pre-defined acceptable level.

As stated in the report, a decision whether or not to repair may be aided by the evaluation of P_f to identify the safe period before repair.

3.4 Appraisal of the limitations of the Analysis

The limitations of the analysis may be summarised below:

1. Whilst good data is available for defining means and COV's of the input variables, the form of the distribution functions are chosen on a less rigorous basis.

2. Stress intensity factors for tubulars are not known with confidence.

3. Interaction effects between defects are not considered.

4. Residual stresses in the HAZ are uncertain.

5. Initial defect sizes are unknown.

6. Acceptable probabilities of failure are not defined rigorously.

7. System reliability analysis needs to be developed to represent the redundancy and load re-distribution present in structures of this type.

Appendix

This is an amplification of Sections 3.3 - 4 of the report, correlation between events Mi and Mj is rigorously examined. It is not thought appropriate to review the details in this document.

4. PROPOSED METHOD

The authors propose that the probabilistic techniques described above be used to more rigorously evaluate the more critical components identified using the deterministic method given in Reference 2 and illustrated in Figure 8.

The points system proposed in Reference 2 involves the evalution of point scores associated with likelihood of failure X and consequences of failure Y. The product formed together with an inspection history parameter gives the inspection weighting index Z.

The probabilistic approach recommended, is illustrated in Figure 9.

Following the methods outlines in Reference 3 and Section 3 the probability of failure of the component P_f should be evaluated. This evaluation will take into account the results of any inspections performed previously. The impact of the existing inspection strategy for component will also be able to be evaluated at this stage. If no inspection results are available then conservative design values for the parameters may be used, see Figure 10. Figure 11 illustrates the evaluation of P_f if inspection results are available.

In principle, it will also be possible to evaluate the probability of failure due to brittle fracture, dropped objects and ship impact with this approach. The consequences of failure branch is somewhat more problematic. The aim of this branch is to evaluate the acceptable probability of failure P_{faccep}.

1. <u>Deterministic Method</u>

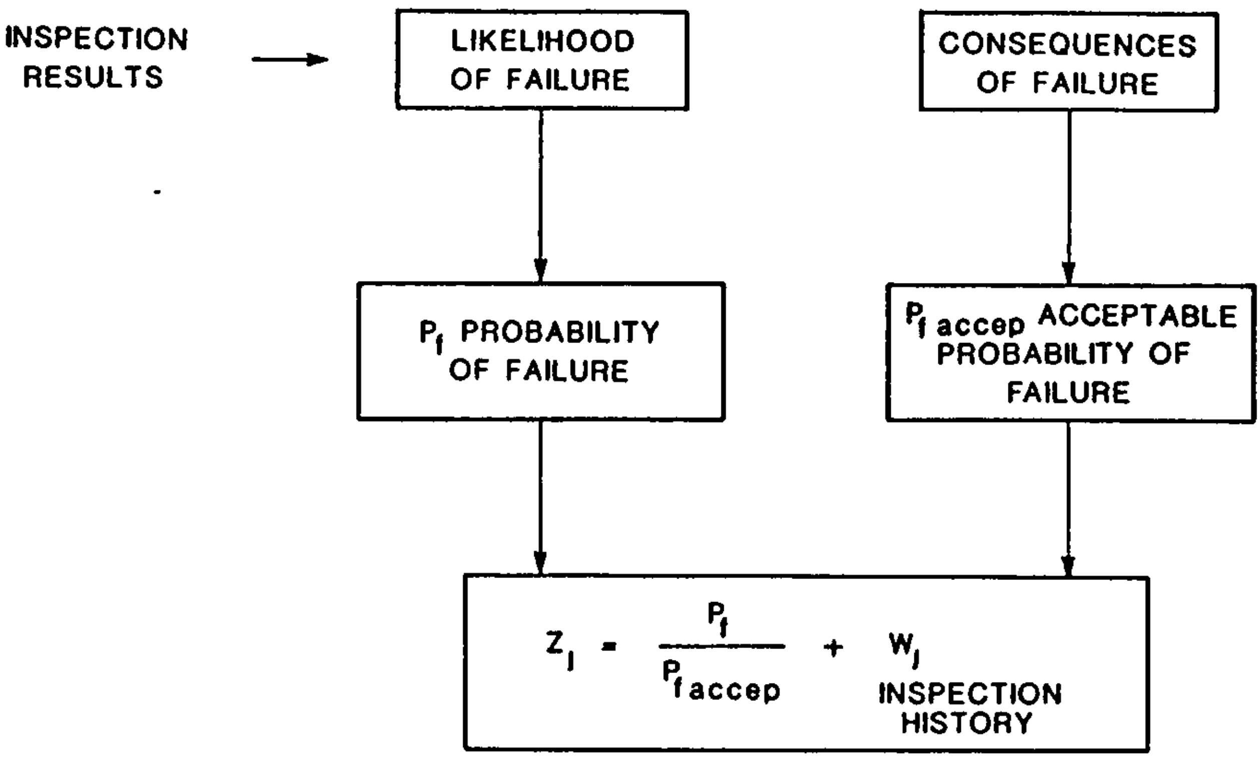
Likelihood of Failure	
Influencing Term	**Rating**
Kj Fatigue Category	
Mj Susceptibility	
Nj Defect Known	
Oj Corrosion Condn.	
Pj Material	
Qj Fabrication Quality	
Rj Excessive Marine Growth	
Sj Confidence in Asst.	
Xj Total	

Consequences of Failure	
Influencing Term	**Rating**
Aj Component Category	
Bj Redundancy (Brace)	
Cj Redundancy (Chord)	
Dj Likelihood Other Losses	
Ej Immediate Risk to Life	
Fj Risk to Environment	
Gj Risk of Lost Production	
Hj Cost of Repair	
Ij Confidence in Asst.	
Yj Total	

Inspection History
Wj

$Zj = Xj \times Yj + Wj$
Inspection Weighting

<u>Figure 8 Computation of Inspection Weighting Z</u>



OVERALL DETERMINATION OF INSPECTION WEIGHTING
PROBABLISTIC METHOD

FIG. 9

Probabilistic Method

1. Likelihood of Failure

a. Without Inspection

Kj Fatigue category (remaining life) from SN curve analysis
Oj Corrosion condn. - worst design
Pj Material Properties - worst in specfication
Qj Fabrication quality - worst in specfication with mismatch SCFs
Rj Marine growth - design levels
Nj Defects - smallest detectable during fabrication

↓

Probabilistic
Fracture Mechanics
Analysis or
Modified S/N Curve

↓

Remaining Life/Failure Category

↓

Probability of Failure in fatigue (or subsequent brittle fracture) ← Mj Susceptibility to dropped objects, ship impact

↓

Pf probability of failure

Figure 10 Likelihood of Failure - Without Inspection

1. Likelihood of Failure

b. With Inspection

(Kj fatigue category a priori)

Oj Corrosion condn. observed

Pj Material properties if available

Qj Fabrication quality observed

Rj Marine growth (affects probability of detection and hence .... crack length)

Nj Defect observed ± uncertainty
If not observed minimum observable ± uncertainty

↓

Probabilistic Fracture
Mechanics Analysis

↓

Remaining life/failure category

↓

Probability of failure ← Mj Susceptibility to failure dropped objects ship impact

↓

$\boxed{\text{Pf probability of failure}}$

Figure 11 Likelihood of Failure - With Inspection

Consequences of Failure

Aj Component category

Bj Redundancy (brace)

Cj Redundancy (chord)

Dj Likelihood other losses

Ej Risk to life

Fj Risk to environment

Gj Risk of lost production

Hj Cost of repair

↓

P_{faccep}	acceptable probability of failure

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5. RECOMMENDATIONS FOR FURTHER WORK

The authors of this review are aware of the dangers in over-complicating the process of the development of an optimised inspection strategy and would suggest the implementation of the points raised in the previous section in such a way that simple procedures be followed in arriving at the inspection weighting Z.

Immediate tasks required are:

The preparation of detailed procedures which describe how the contributing parameters change given the results of an inspection whether or not a defect is observed in a particular location.

An investigation of the deterministic formulation itself in order to firm up on the values and relative values to be assigned for each class of rating, the treatment of confidence as a multiplicative factor and the treatment of W (inspection history weighting).

An investigation of the acceptable probability of failure associated with particular components using a systems approach.

One way of avoiding the over-complication of the model as far as the user is concerned would be to encapsulate the method in a simple to use computer program. A conventional or expert system based approach could be used. The recommended machine would be an IBM PC or compatible.

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